Section 3.1: Linear systems

We learn:

- what is a system of linear equations?
- How to solve one by 'elimination'
- What are elementary operations
- How to find if the system is consistent or inconsistent
- There are 0, 1, or infinitely many solutions

Page 145 question 4.
Solve $5x - 6y = 1$
$6x - 5y = 10$

Solution. We eliminate the variable x

$\frac{6}{5}$ (eqn 1) : $6x - \frac{36}{5}y = \frac{6}{5}$

Subtract this from eqn 2

$0x + (-5 + \frac{36}{5})y = 10 - \frac{6}{5}$

$\frac{11}{5}y = \frac{44}{5}$, $y = 4$

We use 'back substitution' to find x. Substitute the value for y in $5x - 6y = 1$

$x = \frac{1+6y}{5} = 5$

There is a unique solution: (x,y) = (5,4).
We used 'back substitution' and some 'elementary operations'.

Page 145 question 6.

Solve $4x - 2y = 4$
$6x - 3y = 7$

The solutions are (x,y) =

a. (3,2)
b. (2,3)
c. (-1,5)
d. (1/2, -1/3)
e. None of the above. ✓ No solutions.

Add $\left(-\frac{3}{2}\right)$ eqn 1 to eqn 2 :

$0 = 1$

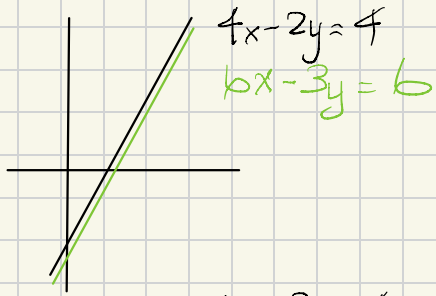
Solve $4x - 2y = 4$
$6x - 3y = 6$

Similar : $0 = 0$, which is OK.

The general solution is: $(x, 2x-2)$
There are infinitely many solutions.

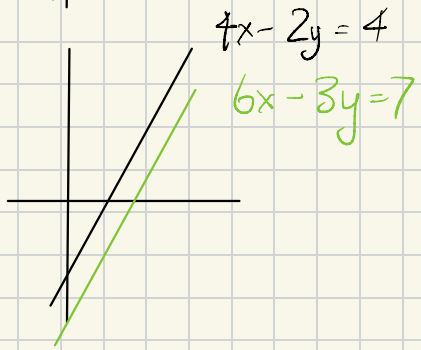
Summary

Solve $4x - 2y = 4$
$6x - 3y = 6$



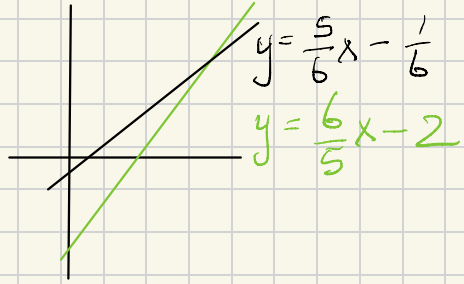
infinitely many solutions

Solve $4x - 2y = 4$
$6x - 3y = 7$



no solutions / inconsistent

Page 145 question 4.
Solve $5x - 6y = 1$
$6x - 5y = 10$



one solution

Elementary operations

1. Multiply an equation by a non-zero scalar.
2. Switch two equations.
3. Add a multiple of one equation to another.

Solve

$$2y + 3z = 7$$
$$2x + 4y + z = -1$$
$$X + 3y + 2z = 3$$

Page 145 question 26.

The equation $y'' - 121y = 0$ has general solution $y = A e^{11x} + B e^{-11x}$.
If $y(0) = 44$ and $y'(0) = 22$, find A and B.